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STUDY ON INCREASING BRAKING PERFORMANCE AND EFFICIENCY OF FOUR-WHEEL-DRIVE TRACTORS THROUGH BRAKING ON ALL WHEELS

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Abstract**—**The paper elaborates a mathematical model with a view to carrying out a study into the dynamics of wheeled tractors during braking. The braking dynamics is analyzed by considering two versions for the braking system: 1) braking applied on the rear axle and 2) braking applied on all four wheels. The mathematical model enables us to graphically illustrate the evolution of the following parameters: braking deceleration, braking speed and the distance traveled by the tractor during braking. The mathematical model elaborated is applied on a 4x4 tractor with engine power of 58 kW.

Keywords**—**braking efficiency, braking performance, tractor.

I. Introduction

We notice that not only in our country but also worldwide there is a tendency to improve tractor's transporting performance by increasing tractor speed. Nevertheless, the increase of the speed at which an agricultural vehicle can be driven calls for special technical restrictions that should enable agricultural vehicles to keep the pace with the other fast vehicles participant in road traffic, taking into account traffic safety.

The main objective of the paper is to underscore the essential advantages of four-wheel braking, grounding the study in the theoretical researches conducted on braking dynamics.

The necessity to brake the tractor and the tractor-trailed appliance system may occur in various situations. Yet, all braking systems are divided into two categories:

1) emergency braking, corresponding to a maximum possible braking intensity;

2) service or partial braking.

The intensity of service braking is always lower than the maximum possible braking. The emergency braking does not exceed 5...10% of the total braking maneuvers; however, this very condition determines the traveling security in critical situations. This is the reason why all the rules and regulations regarding the efficiency of braking systems are established for the emergency braking system.

In order to evaluate the braking performance and efficiency of motor vehicles, the following criteria have been considered:

1) maximum value of braking deceleration $\ddot{x}_{\max}$ reached during braking;

2) braking distance x_F traveled by the tractor from applying the brakes until the vehicle stops;

3) braking time t_F.

Determining the above-mentioned parameters for certain initial conditions and for certain external forces represents one of the aspects of the braking dynamics of tractors and tractor-trailed appliances systems. As results from the *Tractor's theory*, the forces acting on the tractor cannot exceed certain values resulted from the adhesion condition in the rolling system and road contact.

The study of tractor braking dynamics calls for the general principles shaping the dynamics of mechanical systems. The complexity of our study consists is an accurate determination of forces acting on the tractor, i.e. accurate determination of the calculation scheme.

II. TRACTOR BRAKING DYNAMICS

The rectilinear movement of the tractor is analyzed under the conditions of intense braking. The diagram of forces and moments acting during the tractor braking is presented in Fig. 1.

Generally, it is the following actuating forces that reduce the tractor's speed: braking, rolling resistance, air resistance, frictions in the bearings of the rolling system etc. The value of braking forces, under the conditions of a braking of maximum intensity, represents 98% of the total forces that reduce the speed [1]. That is why it is solely the braking forces that are subsequently considered. It is considered that wheel slip is very low. In this case, at any moment, the following condition is met:

$$\omega = \frac{v}{r}, \tag{1}$$

where: ω denotes the wheels angular velocity; v – translational velocities of the wheel's center; r – wheel radius.

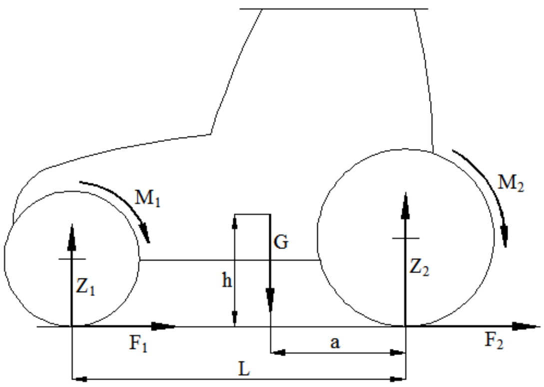

Fig.1. Diagram of forces and moments acting on a 4×4 tractor – braking applied on all wheels

Lagrange equation of second order is used to formulate and solve the equation of motion for the tractor-trailed appliances systems. The system analyzed has a single degree of freedom, determined by the generalized coordinate x [1]–[5].

The kinetic energy of the tractor-trailed appliances system is calculated with the relation:

$$E = \frac{m\dot{x}^2}{2} + \frac{\sum J_i \dot{\theta}_i^{\,2}}{2}, \tag{2}$$

where: m denotes the tractor's mass; J_i – moment of inertia of the tractor's wheel (i = 1 for the front axle, i = 2 for the rear axle);

Considering the relation (1), $\dot{\theta} = \dot{x}/r$, and

$$E = \frac{m_{red}\dot{x}^2}{2}, \tag{3}$$

where: $m_{red} = m + \frac{\sum J_i}{r_i^2}$ – tractor's reduced mass;

The generalized force is determined with the virtual mechanical work. Provided the generalized coordinate increases by δx, the wheel rotates with angle

$$\delta\theta = \frac{\delta x}{r}. \tag{4}$$

The virtual mechanical work of all active forces is calculated for one wheel:

$$\delta L = -M \cdot \delta\theta = -\frac{M}{r}\delta x. \tag{5}$$

The braking force F is not included in the last relation as this force is applied in the instantaneous rotation center of velocities and, subsequently, the mechanical work is zero.

The generalized force of the whole tractor is determined with the following relation:

$$Q = \frac{\delta L}{\partial x} = -\sum \frac{M_i}{r_i}. \tag{6}$$

Tractor's equation of motion is subsequently deduced, using Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{x}}\right) - \frac{\partial E}{\partial x} = Q. \tag{7}$$

Using the relation (2), we obtain:

$$\frac{\partial E}{\partial \dot{x}} = m_{red}\dot{x}; \quad \frac{\partial E}{\partial x} = 0. \tag{8}$$

Thus, the equation will be:

$$\ddot{x} m_{red} = -\sum \frac{M_i}{r_i}. \tag{9}$$

The relation for the tractor braking deceleration is obtained by solving the equation (9) as related to $\ddot{x}$:

$$\ddot{x} = -\frac{1}{m_{red}}\sum \frac{M_i}{r_i} \quad \text{or} \quad \ddot{x} = -\frac{1}{m_{red}}\sum F_i. \tag{10}$$

The braking force applied to tractor's wheels, $F_i = M_i / r_i$, is considered to vary according to an exponential law:

$$\begin{gathered}\sum F_i = F_i(t) = \left(F_{2\max} - fZ_2\right)\left(1 - e^{-\alpha t}\right) + \\ + \left(F_{1\max} - fZ_1\right)\left(1 - e^{-\alpha t}\right) + Gf\end{gathered} \tag{11}$$

where: $F_{i\,\max}$ denotes the maximum braking force for one axle; Z_i – weight distributed on the tractor's braked axle; G – tractor's total weight; f – coefficient of rolling resistance; $\alpha = 6/t_0$ – exponent depending on the time t_0 of full brake application.

Integrating (10) we obtain the equation of tractor's traveling speed:

$$\dot{x} = -\frac{1}{m_{red}}\int_0^t \sum F_i(t)dt + \dot{x}_0, \tag{12}$$

where $\dot{x}_0$ denotes the initial braking speed.

Provided the last relation is equaled to zero, the braking time t_F is obtained until a complete stop.

The distance traveled during braking is obtained by integrating the equation (12):

$$x_F = \int_0^{t_F}\left[\frac{1}{m_{red}}\int_0^{t_F}\sum F_i(t)dt + \dot{x}_0\right]dt. \tag{13}$$

We subsequently analyze the changes in the reactions on tractor's wheel, in case of a stabilized braking system. The balancing equations of the tractor are as follows:

$$\left.\begin{aligned}&Z_1(L-a) - Z_2 a - \left(F_1 + F_2\right)h = 0;\\ &Z_1 + Z_2 = G.\end{aligned}\right\} \tag{14}$$

By solving (10), we obtain:

$$\left.\begin{aligned}Z_1 &= \frac{Ga + Fh}{L};\\ Z_2 &= \frac{G(L-a) - Fh}{L}.\end{aligned}\right\} \tag{15}$$

where $F = F_1 + F_2$.

By replacing in each equation the total braking force with its maximum value, $F_{\max} = \varphi G$, we obtain vertical reactions on the tractor's wheels provided the weight G is fully used. By multiplying the value obtained with the coefficient of road adhesion φ, we obtain the maximum braking force for each axle:

$$\left.\begin{aligned} F_{1\max} &= \frac{G(a+h\varphi)}{L}\varphi; \\ F_{2\max} &= \frac{G(L-a-h\varphi)}{L}\varphi. \end{aligned}\right\} \quad (16)$$

Tractors from class 6…14 kN usually have solely rear axle brakes. The vertical reaction on the rear axle may be similarly determined from the equation of moments in relation to tractor's center of gravity

$$\begin{aligned} &Z_1(L-a) - Z_2 a - F_2 h = 0 \quad \Rightarrow \\ &\Rightarrow (G - Z_2)(L-a) - Z_2 a - F_2 h = 0, \end{aligned} \quad (17)$$

resulting

$$Z_2 = \frac{G(L-a) - F_2 h}{L}. \quad (18)$$

By replacing in this relation the braking force F_2 with its maximum value, $F_{\max} = Z_2\varphi$, we obtain:

$$Z_2 = \frac{G(L-a) - Z_2 h\varphi}{L}. \quad (19)$$

By solving the last equation in relation to the vertical reaction Z_2, we obtain:

$$Z_2 = \frac{G(L-a)}{L + h\varphi}. \quad (20)$$

Consequently, the maximum braking force in case of a tractor with rear axle braking will be:

$$F_{\max} = \frac{G(L-a)\varphi}{L + h\varphi}. \quad (21)$$

III. APPLICATIONS OF THE ESTABLISHED MATHEMATICAL MODEL

Using the above-written mathematical model, we present the results obtained for the braking performance of 4x4 tractors considering two braking systems versions: 1) braking applied on the rear axle and 2) braking applied on all four wheels. As exemplification, we used the wheeled tractor MAT 81 (manufacturer MAT Craiova) with power of 58 kW (78.9 HP). We mention that the tractor submitted to analysis is braked solely on rear axle wheels, and the front axle is coupled through an electro-hydraulic system. This construction enables, by means of simple procedures, the synchronization between braking application on rear wheels and front axle coupling. In this way, the kinematic link between the two axles allows the braking to be applied on all wheels. This procedure is required only in case of high traveling speed.

For the mathematical model we considered the following input data: G = 48.95 kN (value used for tractor braking testing on concrete track)); Z_1 = 21.73 kN – load on the front axle; Z_2 = 27.22 kN – load on the rear axle; L = 2530 mm –wheel base; h = 847 mm –height of center of gravity of the tractor; v_0 = $\dot{x}_0$ = 40.2 km/h – initial braking speed; f = 0.02 –coefficient of rolling resistance; φ = 0.4, 0.5, 0.6, 0.7, 0.8 – coefficient of road adhesion.

The output data take the form of the main braking parameters: braking deceleration, braking time and braking distance.

The results of the analytical calculation are graphically presented for both braking versions (Fig. 2…7).

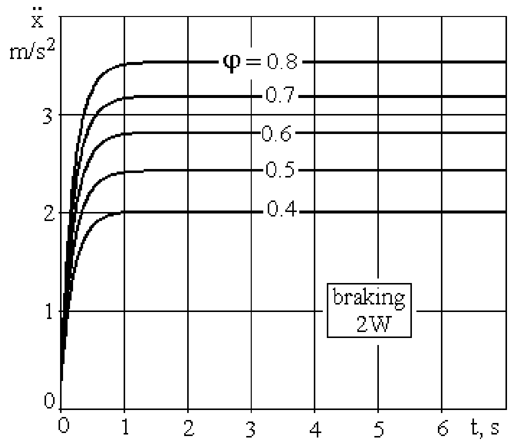

Fig. 2. Variation of tractor deceleration during braking taking into account the time, for 5 values of the coefficient of road adhesion – braking applied on rear axle wheels.

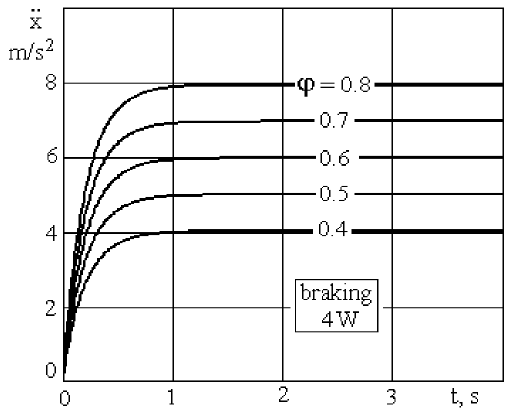

Fig. 3. Variation of tractor deceleration during braking taking into account the time, for 5 values of the coefficient of road adhesion – braking applied on all wheels.

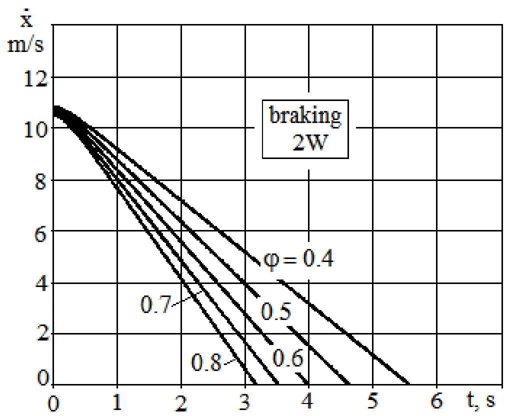

Fig. 4. Variation of tractor speed during braking taking into account the time, for 5 values of the coefficient of road adhesion – braking applied on rear axle wheels.

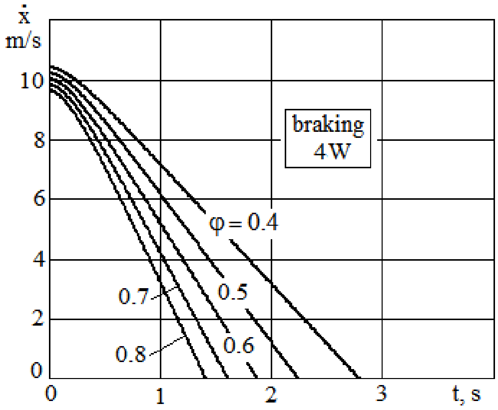

Fig. 5. Variation of tractor speed during braking taking into account the time, for 5 values of the coefficient of road adhesion – braking applied on all wheels.

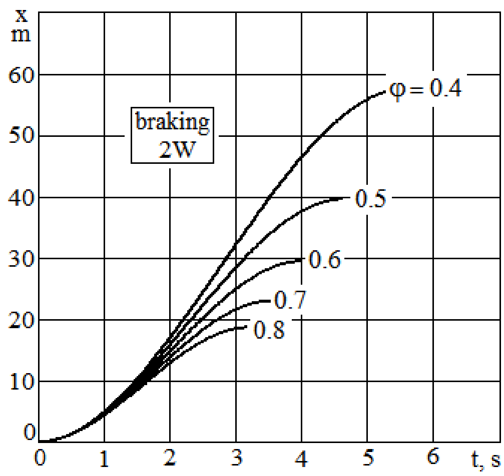

Fig. 6. Variation of distance covered by the tractor during braking taking into account the time to a complete stop, for 5 values of the coefficient of road adhesion – braking applied on rear axle wheels.

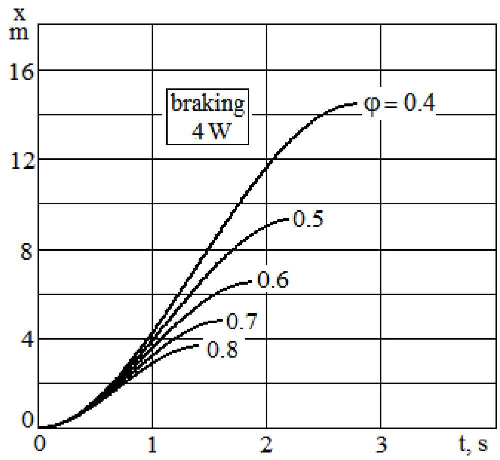

Fig. 7. Variation of distance covered by the tractor during braking taking into account the time to a complete stop, for 5 values of the coefficient of road adhesion – braking applied on all wheels.

The table presents the values of maximum deceleration of both the time and distance braking. Likewise, for discussion and comparison purposes, the values obtained for MAT 81 tractor braking testing on concrete track are also presented [6].

TABLE I
BRAKING PARAMETERS FOR TRACTOR MAT 81 IN TWO VERSIONS FOR THE BRAKING SYSTEM

Coefficient of road adhesion φ	Braking version	Parameters		
		$\ddot{x}_{max}$ m/s^2	t_F s	x_F m
0.4	2W	2.01	5.55	57.38
	4W	4.01	2.78	14.40
0.5	2W	2.42	4.61	39.52
	4W	4.99	2.23	9.28
0.6	2W	2.81	3.97	29.33
	4W	5.97	1.87	6.46
0.7	2W	3.18	3.51	22.93
	4W	6.95	1.60	4.75
0.8	2W	3.53	3.16	18.62
	4W	7.93	1.40	3.63
Experimental tests on asphalt road [6]	2W	3.20*	–	19.50

* Average deceleration
Notation: 2W – two-wheel braking; 4W – four-wheel braking

IV. CONCLUSIONS

The mathematical model elaborated is general. It allows the determination of braking parameters for all types of wheeled tractors (4×2 and 4×4) and for various types of soil, characterized through both the coefficient of road adhesion and the coefficient of rolling resistance. Similarly, the mathematical model enables us to carry out analyses into the braking system performance considering the two braking systems versions: 1) braking applied on the rear axle and 2) braking applied on all four wheels.

For a particular braking version, the braking parameters depend on the initial braking speed and on the coefficient of road adhesion. When brakes are applied on all four wheels the braking performance parameters are considerably improved: the maximum deceleration increases by more than twice as much whereas the braking time decreases by more than twice; the braking distance also decreases by 4...5 times. The results of the theoretical researches are very similar to those obtained for tractor braking testing on concrete track.

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